FUZZY ASSESSMENT OF THE "5 E'S" INSTRUCTIONAL TREATMENT FOR TEACHING MATHEMATICS TO ENGINEERING STUDENTS

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ABSTRACT

The "5 E's" instructional treatment, which is based on the principles of social constructivism, is currently a very popular method for teaching, especially in school education. A hybrid model is developed in the present paper for assessing the effectiveness of the "5 E's" application for teaching mathematics to engineering students of the University of Peloponnese, Greece. The model uses grey numbers and neutrosophic sets for evaluating the mean student performance, whereas the quality performance is assessed by calculating the Grade Point Average index.

KEYWORDS

Fuzzy Sets, Neutrosophic Sets, Grey Numbers, Fizzy Assessment, "5 E's" Instructional Treatmen, Grade Point Average (GPA) index.

1. INTRODUCTION

The idea that knowledge is a human construction supported by the experience, which was first stated by Vico in the 18th century and was further extended by Kant, affected greatly the epistemology of Piaget, who is considered to be the forerunner of the theory of *constructivism* for the process of learning. Constructivism introduced formally by von Clasersfeld in Piaget's foundation of the USA in 1975 [1]. The constructivist approach is based on the following two principles:

- Knowledge is not passively received from the environment, but it is actively constructed by synthesizing past knowledge and experience with the new information.
- The "coming to know" is a process of adaptation based on and constantly modified by the individual's experience of the world.

On the other hand, the *socio-cultural* theory for learning is based on Vygotsky's ideas claiming that knowledge is a product of culture and social interaction. Learning takes place when the individuals engage socially to talk and act about shared problems or interests [2-4].

The combination of the principles of constructivism with the socio-cultural ideas created the theory of *social constructivism* for learning [3, 5]. The *"5 E's"* is an instructional model for teaching based on the principles of social constructivism, which has become very popular recently, especially in school education. Each of the "5 E's" describes a phase of learning which begins with the letter "E" [6].

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In this paper a fuzzy assessment method is developed for evaluating the effectiveness of the "5 E's" instruction for teaching mathematics to engineering students, which uses *grey numbers (GNs) and neutrosophic sets (NSs)* as tools. In particular, this method is very useful when the instructor is not sure about the grades assigned to each student for evaluating his/her individual performance.

The rest of the paper is organized as follows: Section 2 includes a brief description of the "5 E's" instructional treatment. Section 3 introduces the mathematical background about GNs and NSs being necessary for the understanding of the paper. The assessment method with GNs and NSs is developed in Section 4 and it is applied for evaluating the effectiveness of the "5 E's" instructional treatment in Section 5. The article closes with the final conclusions and a brief discussion for further research, which are presented in Section 6.

2. THE "5 E'S INSTRUCTIONAL TREATMENT

The consecutive phases of the "5 E's instructional treatment are the following:

- *Engage (E_1):* This is the starting phase which connects the past with the present learning experiences and focuses student thinking on the learning outcomes of the current activities.
- *Explore (E_2):* During this phase students explore their environment to create a common base of experiences by identifying and developing concepts, processes and skills.
- *Explain (E_3):* In this phase students explain and verbalize the concepts that they have been explored and they develop new skills. The teacher has the opportunity to introduce formal terms, definitions and explanations for the new concepts and processes and to demonstrate new skills or behaviors.
- *Elaborate (E_4):* In this phase students develop a deeper and broader conceptual understanding and obtain more information about areas of interest by practicing on their new skills and behaviors.
- *Evaluate (E_5):* This is the final step of the "5E's" instructional model, where learners are encouraged to assess their understanding and abilities and teachers evaluate student skills on the new knowledge.

Depending on the student reactions, there are forward or backward transitions between the three middle phases (explore, explain, elaborate) of the 5E's model during the teaching process (see Figure 1).

Figure 1: The flow diagram of the "5 E's instructional treatment

The "5 E's" model allows students and teachers to experience common activities, to use and build on prior knowledge and experience and to assess their understanding of a concept. Although it has been mainly applied in school education [7], it can be used with students of all ages, including adults [8].

3. MATHEMATICAL BACKGROUND

3.1. Fuzzy Sets and Logic

The development of human science and civilization owes a lot to Aristotle's (384-322 BC) *bivalent logic (BL)*, which was in the center of human reasoning for centuries. BL is based on the *"Principle of the Excluded Middle",* according to which each proposition is either true or false. Opposite views, however, appeared also early in the human history supporting the existence of a third area between true and false, where these two notions can exist together; e.g. by Buddha Siddhartha Gautama (India, around 500 BC), by Plato (427-377 BC), more recently by the Marxist philosophers, etc. Integrated propositions of multi-valued logics reported, however, only during the early 1900's, mainly by Lukasiewicz and Tarski [9, Section 2]. According to the Lukasiewicz's *"Principle of Valence"* propositions are not only either true or false, but they may have intermediate truth-values too.

Zadeh, replacing the characteristic function of a crisp subset of the universe U with the *membership function* m: U→ [0, 1], introduced in 1965 the concept of *fuzzy set (FS)* [10], in which each element x of U has a membership degree m(x) in the unit interval. The closer m(x) to 1, the better x satisfies the characteristic property of the corresponding FS. For example, if A is the FS of the tall men of a country and m(x) = 0.8, then x is a rather tall man. On the contrary, if m(x) = 0.4, then x is a rather short man. Formally, a FS A in U can be written as a set of ordered pairs in the form

$$F = \{(x, m(x)): x \in U\} \quad (1)$$

As an example, Figure 2 represents the graph of the FS T of "tall people". People with heights less than 1.50 m are considered of having membership degree 0 in T. The membership degree is continuously increasing for heights greater than 1.50m, taking its maximal value 1 for heights equal or greater than 1.80 m. Therefore, the "fuzzy part" of the graph - which is conventionally represented in Figure 2 by the straight line segment AC, but its exact form depends upon the way in which the membership function has been defined - lies in the area of the rectangle ABCD defined by the OX axis, its parallel through the point E and the two perpendicular to it lines at the points A and B.

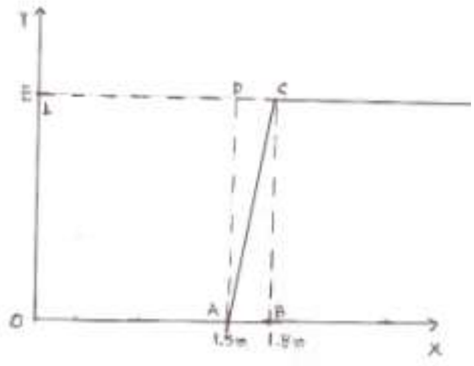

Figure 2: Graphical representation of the FS of "tall people"

Zadeh also introduced, with the help of FS, the infinite-valued in the unit interval *fuzzy logic (FL)* [11], on the purpose of dealing with the existing in the everyday life partial truths. FL, in which truth values are modelled by numbers in the unit interval, embodies the Lukasiewicz's "Principle of Valence".

Uncertainty can be defined as the shortage of precise knowledge or complete information on the data that describe the state of a situation. It was only in a second moment that FS theory and FL were used to embrace uncertainty modelling. This happened when membership functions were reinterpreted as possibility distributions [12, 13]. Zadeh [12] articulated the relationship between possibility and probability, noticing that what is probable must preliminarily be possible.

Probability theory used to be for a long period the unique tool in hands of the specialists for dealing with problems connected to uncertainty. Probability, however, was proved to be sufficient only for tackling the cases of uncertainty which are due to *randomness* [14]. Randomness characterizes events with known outcomes which, however, cannot be predicted in advance, e.g. the games of chance. FSs, apart from randomness, tackle also successfully the uncertainty due to *vagueness*, which is created when one is unable to distinguish between two properties, such as “a good player” and “a mediocre player”. For general facts on FSs and the connected to them uncertainty we refer to the book [15].

3.2. Neutrosophic Sets

Several generalizations and extensions of the theory of FSs have been developed during the last years for the purpose of tackling more effectively all the forms of the existing in real world uncertainty. The most important among them are briefly reviewed in [16].

Atanassov in 1986, considered, in addition to Zadeh’s membership degree, the degree of *non-membership* and extended FS to the notion of *intuitionistic FS (IFS)* [17]. Smarandache in 1995, inspired by the frequently appearing in real life neutralities - like <friend, neutral, enemy>, <win, draw, defeat>, <high, medium, short>, etc. - generalized IFS to the concept of *neutrosophic set* (NS) by adding the degree of *indeterminacy* or *neutrality* [18]. The word “neutrosophy” is a synthesis of the word “neutral′ and the Greek word “sophia” (wisdom) and means “the knowledge of the neutral thought”. The simplest form of a NS is defined as follows:

Definition 1: A *single valued NS (SVNS)* A in the universe U is of the form

$$A = \{(x,T(x),I(x),F(x)): x \in U,\ T(x),I(x),F(x) \in [0,1],\ 0 \leq T(x)+I(x)+F(x) \leq 3\} \quad (2)$$

In equation (2) T(x), I(x), F(x) are the degrees of *truth* (or membership), *indeterminacy* (or neutrality) and *falsity* (or non-membership) of x in A respectively, called the *neutrosophic components* of x. For simplicity, we write A<T, I, F>. Indeterminacy is defined to be in general everything that exists between the opposites of truth and falsity [19].

Example 1: Let U be the set of the players of a soccer club and let A be the SVNS of the good players of the club. Then each player x is characterized by a *neutrosophic triplet* (t, i, f) with respect to A, with t, i, f in [0, 1]. For example, x(0.7, 0.1, 0.4) ∈ A means that there exists a 70% belief that x is a good player, but at the same time there exist a 10% doubt about it and a 40% belief that x is not a good player. In particular, x (0, 1, 0) ∈ A means that we do not know absolutely nothing about the quality of player x (new player).

If the sum T(x) + I(x) + F(x) < 1, then it leaves room for incomplete information about x, if it is equal to 1 for complete information and if it is >1 for *inconsistent* (i.e. contradiction tolerant) information about x. A SVNS may contain simultaneously elements leaving room to all the previous types of information. All notions and operations defined on FSs are naturally extended to SVNSs [20].
Summation of neutrosophic triplets is equivalent to the union of NSs. That is why the neutrosophic summation and implicitly its extension to neutrosophic scalar multiplication can be

defined in many ways, equivalently to the known in the literature neutrosophic union operators [21]. For the needs of the present work, writing the elements of a SVNS A in the form of neutrosophic triplets and considering them simply as *ordered triplets* we define addition and scalar product as follows:

Definition 2: Let (t_1, i_1, f_1), (t_2, i_2, f_2) be in A and let k be a positive number. Then:

- The *sum* $(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1 + t_2, i_1 + i_2, f_1 + f_2)$ (2)
- The *scalar product* $k(t_1, i_1, f_1) = (kt_1, k i_1, kf_1)$ (3)

Remark 1: Summation and scalar product of the elements of a SVNS A with respect to Definition 2 *need not* be closed operations in A, since it may happen that $(t_1 + t_2)+(i_1 + i_2)+(f_1 + f_2)>3$ or $kt_1+k i_1 + kf_1>3$. With the help of Definition 2, however, one can define *in A* the *mean value* of a finite number of elements of A as follows:

Definition 3: Let A be a SVNS and let (t_1, i_1, f_1), (t_2, i_2, f_2), …., (t_k, i_k, f_k) be a finite number of elements of A. Assume that (t_i, i_i, f_i) appears n_i times in an application, i = 1,2,…., k. Set $n = n_1+n_2+\ldots+n_k$. Then the *mean value* of all these elements of A is defined to be the element (t_m,i_m,f_m) of A calculated by

$$\frac{1}{n}[n_1(t_1, i_1,f_1)+n_2(t_2,i_2,f_2)+\ldots+n_k(t_k,i_k, f_k)] \quad (4)$$

3.3. Grey Numbers

The theory of *grey systems* [22] introduces an alternative way for managing the uncertainty in case of approximate data. A grey system is understood to be any system which lacks information, such as structure message, operation mechanism or/and behavior document.

Closed real intervals are used for performing the necessary calculations in grey systems. In fact, a closed real interval [x, y] could be considered as representing a real number T, termed as a GN, whose exact value in [x, y] is unknown. We write then $T \in [x, y]$. A GN T, however, is frequently accompanied by a *whitenization function* $f: [x, y] \to [0, 1]$, such that, if f(a) approaches 1, then a in [x, y] approaches the unknown value of T. If no whitenization function is defined, it is logical to consider as a representative crisp approximation of the GN T the real number

$$V(T) = \frac{x+y}{2} \quad (3)$$

The arithmetic operations on GNs are introduced with the help of the known arithmetic of the real intervals [23]. In this work we are going to make use only of the addition of GNs and of the scalar multiplication of a GN with a positive number, which are defined as follows:

Definition 2: Let $A \in [x_1, y_1]$, $B \in [x_2, y_2]$ be two GNs and let k be a positive number. Then:

- The sum: A+B is the GN $A+B \in [x_1+y_1, x_2+y_2]$ (4)
- The scalar product kA is the GN $kA \in [kx_1, ky_1]$ (5)

4. FUZZY ASSESSMENT METHODS WITH QUALITATIVE GRADES

In many cases it is a common practice to assess the student performance by using qualitative (linguistic) instead of numerical grades. A widely accepted scale of such grades is the following: A=excellent, B=very good, C=good, D=mediocre and F=unsatisfactory. Here we present three fuzzy assessment methods that we are going to use in this work for assessing the overall performance of a student group.

4.1. Mean Performance

In case of using qualitative grades the mean performance of a student group cannot be evaluated with the classical method of calculating the mean value of the student individual grades. To overcome this difficulty, we assign to each grade a GN, denoted for simplicity with the same letter, as follows: A = [85, 100], B = [75, 84], C = [60, 74], D = [50, 59], F = [0, 49]. The choice of the above GNs, although it corresponds to generally accepted standards, is not unique. For example, for a more strict assessment, one may choose A= [90, 100], B = [80, 89], C= [70, 79], D = [60, 69], F = [0, 59], etc. Such changes, however, does not affect the generality of our method

Assume now that, from the n in total students of the group, n_X obtained the grade X=A, B, C, D, F. It is logical then to accept that the crisp approximationV(M) of the GN

$$M = \frac{1}{n}(n_A A + n_B B + n_C C + n_D D + n_F F) \quad (5)$$

Can be used for estimating the mean performance of the student group.

4.2. Quality Performance

A very popular in the USA and other countries method for evaluating the quality performance of a group is the use of the *Grade Point Average (GPA)* index [24, p.125], which is calculated by the formula

$$GPA = \frac{0n_F + n_D + 2n_C + 3n_B + 4n_A}{n} \quad (6)$$

In other words, the GPA index is a weighted average in which greater coefficients (weights) are assigned to the higher grades. Note that, since in the worst case ($n=n_F$) is GPA=0 and in the ideal case ($n=n_A$) is GPA=4, we have in general that

$$0 \leq GPA \leq 4 \quad (7)$$

When two groups have the same GPA index, however, this method is not sufficient to show which of them performs better. In such cases the *Rectangular Fuzzy Assessment Model (RFAM)*, which is based on the *Center of Gravity (COG) defuzzification technique* can be used [24, pp. 126-130].

4.3. Neutrosophic Assessment

Frequently in practice the instructor has doubts about the grades assigned to some students, either because he/she had not the opportunity to evaluate their skills explicitly during a course, or because they didn't clarify their answers properly in a written test. In such cases, the most suitable method for assessing the overall performance of a student group is to use NSs as tools. Considering, for example, the NS of the good students of the group, one introduces neutrosophic triplets characterizing the individual performance of each student and then calculates the mean value of all these triplets with the help of equation (4) in order to obtain the proper conclusions about the group's overall performance. In order to have complete information for each student's performance, the sum of the component of each triplet must be equal to 1.

5. THE CLASSROOM APPLICATION

The purpose of the following classroom application was to evaluate the effectiveness of the "5 E's" instructional treatment for teaching mathematics to engineering students. The subjects were the first term students of two departments of the School of Engineering of the University of Peloponnese during the teaching of the course "Higher Mathematics I", which includes Complex Numbers, Differential and Integral Calculus in one variable and elements from Linear Algebra. According to the grades obtained in the PanHellenic examination for entrance in Higher Education, the potential of the two departments in mathematics was about the same. The course's instructor was also the same person, but the teaching methods followed were different. Namely, the "5 E's" approach was applied for teaching the course to the 60 students of the first department (experimental group), whereas the classical method with lectures and exercises on the board was applied for the 60 students of the second department (control group).

The results of the final examination, after the end of the course, were the following:

- Department I: A: 9 students, B: 15, C: 18, D: 12, F: 6
- Department II: A: 12, B: 15, C: 9, D: 12, F: 12

Therefore, applying the assessment methods of Section 4, we evaluated the performance of the two departments as follows:

Mean performance

By equation (5) one finds that

$$M_I = \frac{1}{60}\,(9[85,100]+15[75,84]+18[60,74]+12[50,59]\ +6[0,49]) = \frac{1}{60}[3570,4994] \approx [59.5,83.23].$$

Therefore, equation (3) gives that $V(M_I)\approx 71.36$, which shows that the experimental group demonstrated a good (C) mean performance.

In the same way one finds that $V(M_{II})\approx 62.56$, which shows that the control group also demonstrated a good (C) mean performance, which, however, was 8.8% worse than that of the experimental group.

Quality performance

Equation (6) gives that

$GPA_I = \dfrac{12+2*18+3*15+4*9}{60} = 2.12$ and similarly $GPA_{II}=2.05$, which shows that the experimental group demonstrated a slightly better quality performance. In fact, with the help of equation (7) it is easy to check that the superiority of the experimental group in this case is only 0.07*25=1.75%.

Note that, some of the student answers in the final examination were not clearly presented or well justified. As a result, the instructor was not quite sure for the accuracy of the grades assigned to them. For this reason, we decided to apply the neutrosophic method of section 4.3 for the assessment of the two departments' overall performance. For this, starting from the students with the higher grades, let us denote by S_i, i=1,2,….,60, the students of each department. Considering the NS of the good students, we assigned neutrosophic triplets to all students of the two departments as follows:

- Department I: S_1-S_{32}: (1,0,0), S_{33}-S_{38}: (0.8,0.1,0.1), S_{39}-S_{42}: (0.7,0.2,0.1), S_{43}-S_{46}: (0.4,0.2,0.4), S_{47}-S_{50}: (0.3,0.2,0.5), S_{51}-S_{53}: (0.2,0.2,0.6), S_{54}-S_{55}: (0.1,0.2,0.7), S_{56}-S_{57}: (0,0.2,0.0.8), , S_{578}-S_{60}: (0,0,1).
- Department II: S_1-S_{31}: (1,0,0), S_{32}-S_{35}: (0.8,0.1,0.1), S_{36}: (0.7,0.1,0.2), S_{35}-S_{43}: (0.4,0.1,0.5), S_{44}-S_{46}: (0.3,0.2,0.5), S_{47}-S_{50}: (0.2,0.2,0.6), S_{51}-S_{52}: (0.1,0.2,0.7), S_{53}-S_{58}: (0,0.3,.0.7), , S_{59}-S_{60}: (0,0,1).

Then, by equation (4), the mean value of the neutrosophic triplets of Department I is equal to $\frac{1}{60}$[32(1,0,0)+6 (0.8,0.1,0.1)+ 4(0.7,0.2,0.1)+4 (0.4, 0.2,0.4)+ 4(0.3,0.2,0.5)+3(0.2,0.2,0.6)+ 2(0.1,0.2, 0.7)+2(0,0.2,0.0.8)+3 (0,0,1)≈(0.72, 0.07, 0.21). In the same way one finds that the mean value of the neutrosophic triplets of Department II is equal to (0.65, 0.08, 0.27).

Thus, the probability for a random student of Department I to be a good student is 72%, but at the same time there exists a 7% doubt about it and a 21% probability to be not a good student. Also, the probability for a random student of Department II to be a good student is 65%, with a 8% doubt about it and a 27% probability to be not a good student. Consequently, the experimental group, despites the doubts of the instructor for the grades assigned to the students, demonstrated a better overall performance.

6. CONCLUSION

The classroom application presented in this work demonstrated a superiority of the experimental group with respect to the control group. This superiority was significant concerning the two groups' mean and overall (in terms of the neutrosophic assessment) performance, but rather negligible concerning their quality performance. This gives a strong indication that the application of the "5 E's" method benefits more the mediocre and the weak in mathematics students, but less the good students. Much more experimental research is needed, however, for obtaining safer conclusions. Other applications of FSs and their extensions is of course another interesting topic for future research, e.g. see {25, 26], etc.

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